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The Effect of Virtual Manipulatives on Senior Secondary School Students' Achievement, Actual Understanding and Perceived Understanding in Geometry

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Abstract

This study examined the effect of Virtual Manipulatives on Senior Secondary School One students' achievement, actual understanding, and perceived understanding in Geometry. A quasi-experimental, pretest–posttest, non-equivalent control group design was adopted. Three co-educational schools were purposively selected. A total of 240 SS1 students participated: 120 in the Virtual Manipulatives group [66 males, 54 females] and 120 in the Traditional Method group. Data were collected using the Achievement Test [KR-20 = .83], Actual Understanding Test in Geometry [$\alpha = .84$], and Virtual Manipulative Understanding Perception Scale [$\alpha = .81$]. The experimental group was taught with GeoGebra applets for six weeks, two 40-min periods per week, while the control group received conventional instruction. Data were analyzed using means, standard deviations, and two-way ANCOVA at $\alpha = .05$. Results revealed that students taught with Virtual Manipulatives had significantly higher posttest scores in achievement, $F(1, 235) = 143.01$, $p < .001$, $\eta^2 = .378$; actual understanding, $F(1, 235) = 120.47$, $p < .001$, $\eta^2 = .339$; and perceived understanding, $F(1, 235) = 89.62$, $p < .001$, $\eta^2 = .276$, than the control group. There was no significant main effect of gender, nor interaction effect of method and gender on any outcome, $p > .05$. It was concluded that Virtual Manipulatives significantly improved students' achievement, actual understanding, and perceived understanding in Geometry, with benefits consistent across gender. It was recommended that Mathematics teachers should integrate VM tools into instruction and that government should provide digital infrastructure in schools.

Keywords: Virtual Manipulatives, GeoGebra, Geometry, Achievement, Actual Understanding, Perceived Understanding, Gender

I. Introduction

A virtual manipulative [VM] is defined as an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge (Moyer, Bolyard, & Spikell, 2002). VMs have also been defined as computer-based renditions of common mathematics manipulatives and tools (Doward, 2002). Zbiek, Heid, Blume & Dick (2007) collectively described VMs as cognitive technological tools. Goldin (2003) conceptualized VMs as a unique externalized representational form. Because a representation is a configuration of signs, characters, icons, or objects that stand for something else, VMs may be considered a unique form of representation or a combination of several representations. Students' capacity to translate among multiple representational systems influences their ability to model and understand mathematical constructs (Goldin & Shteingold, 2001). VMs are often dynamic visual/pictorial replicas of physical manipulatives, such as pattern blocks, base-10 blocks, geometric solids, tangrams, or geoboards. They are placed on the Internet as applets, or smaller stand-alone versions of application programs. Users manipulate these dynamic, visual objects with a computer mouse. This capacity for manipulation makes VMs particularly useful for interactive mathematics instruction. As cognitive tools, VMs allow users to act on virtual objects as representations, with the consequences of those actions producing visual on-screen feedback. Although VMs share similarities with physical manipulatives, they possess unique characteristics that extend beyond the capabilities of physical tools. Their potential for mathematically meaningful actions by users may therefore influence learning. For example, some VMs include links among enactive, iconic, and symbolic notations, thereby supporting learners in making connections among these forms of representation. Other VMs can be altered, including changing the shape of the on-screen object or marking the object with mathematical notations. In addition, VMs are readily available, with unlimited access to many copies of an electronic object via a mouse click (Moyer, Bolyard, & Spikell, 2001; Moyer, Niezgoda, & Stanley, 2005). When selecting a VM for instructional use, it is important to consider mathematical fidelity, cognitive fidelity, pedagogical fidelity, and externalized representations of each tool or applet (Zbiek et al., 2007). Mathematical fidelity refers to the degree to which the mathematical object is faithful to the underlying mathematical properties of that object in the virtual environment. Cognitive fidelity refers to how well the virtual tools reflect the user's cognitive actions and possible choices while using the tool. Pedagogical fidelity is "the extent to which teachers [as well as students] believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teacher's practice" [Zbiek et al., 2007, p. 3].

Geometry is a core strand of the Senior Secondary School Mathematics curriculum in Nigeria. It develops students' spatial reasoning, logical thinking, and problem-solving skills [Federal Ministry of Education, 2013]. However, students' performance in Geometry at the West African Senior School Certificate Examination [WASSCE] has remained consistently low. Chief Examiners' reports have repeatedly cited students' inability to understand concepts, apply formulas, and transfer knowledge to novel problems (WAEC, 2021; 2022; 2023). One reason for this poor performance is the continued use of traditional, teacher-centered methods that rely on chalkboard illustrations and rote memorization of formulas (Yusuf, Afolabi & Loto, 2020). Such methods do not allow students to visualize, manipulate, or explore geometric relationships actively. In response, educators have advocated for the use of technology, specifically VMs. VMs are interactive, web-based visual representations of mathematical objects that students can manipulate to discover concepts (Moyer-Packenham & Bolyard, 2017). Tools such as GeoGebra allow students to drag vertices, cut and reassemble shapes, and observe dynamic changes in real time. While studies have reported positive effects of VMs on achievement (Ozdemir & Sahin, 2021), there is limited evidence from Nigeria on their impact on both actual understanding and perceived understanding within a multi-school design. This study therefore compared VM instruction with traditional methods on SS1 students' achievement, actual geometric understanding, and perceived understanding in Oshimili South Local Government Area, Delta State.

II. Literature Review

This study is underpinned by three theories. First, Cognitive Load Theory [CLT; Sweller, 1988] which posits that learning is influenced by three types of cognitive load: intrinsic, extraneous, and germane. CLT suggests that managing cognitive load enhances learning. VMs in Geometry can reduce extraneous load by providing interactive visualizations, thereby focusing students' attention on geometric relationships. Tools such as GeoGebra can help students manage complexity in geometric concepts. Second, Dual Coding Theory [DCT; Paivio, 1986]. This essentially posits that information is processed via verbal and visual channels, and that dual verbal and visual processing aids learning. VMs provide dynamic visual representations that complement verbal explanations in Geometry. By leveraging visual-spatial processing, VMs may aid Geometry understanding. Third, Mayer's (2009) Cognitive Theory of Multimedia Learning posits that learning is enhanced when learners build mental models through active manipulation of visual and verbal representations. VMs can reduce extraneous cognitive load and promote germane load, leading to deeper understanding [Fyfe et al., 2022].

VMs are digital counterparts of physical objects (Suh & Moyer-Packenham, 2022). Unlike static diagrams, VMs are dynamic and interactive. Actual understanding refers to demonstrable competence, while perceived understanding refers to learners' self-efficacy (Moyer-Packenham, 2017). Ozdemir and Sahin (2021) found that GeoGebra improved seventh-grade students' Geometry achievement. Yusuf et al. (2020) reported similar results in Ilorin. However, few studies have used a three-parameter outcome measure in a multi-school Nigerian context, which this study addresses. Research has increasingly focused on VMs to enhance understanding of abstract concepts such as Geometry, which is critical for developing spatial reasoning and problem-solving skills. A meta-analysis of 66 research reports by Moyer-Packenham and Westenskow (2013) found that VMs had a moderate effect on student achievement compared to other instructional treatments. Comparative studies examining VMs versus traditional methods [TM] or physical manipulatives show mixed but promising results. Drickey (2000) found that middle school students using VMs for spatial reasoning tasks showed positive outcomes. Reimer and Moyer (2005) explored third graders learning fractions with VMs and noted that their dynamic nature facilitated exploration. Moyer-Packenham, Salkind, and Bolyard (2016) analyzed lesson summaries of K–8 teachers' use of VMs and found that tangible manipulatives can lead to improved mathematical performance, particularly in tasks such as number composition. Research in Nigerian contexts suggests that instructional scaffolding and concrete manipulative strategies significantly improve upper basic students' Geometry achievement compared to conventional methods [Yusuf et al., 2020]. Gender influences on VM effectiveness show mixed findings; some research indicates that instructional manipulatives enhance performance equally for male and female students. Contextual factors such as school setting, technology infrastructure, and teacher beliefs are critical considerations (Ngozi, 2010). Although research on VMs is still developing, classroom studies and dissertations have demonstrated unique features of these tools for teaching mathematics. Overall, studies have indicated that students using VMs, either alone or in combination with physical manipulatives, demonstrate gains in mathematics achievement and understanding (Bolyard, 2006; Moyer et al., 2005; Reimer & Moyer, 2005; Suh, 2005; Suh & Moyer, 2007; Suh, Moyer, & Heo, 2005] and appear to be more engaged and on task (Drickey,2000).

Despite its importance, Geometry remains a challenging subject for many secondary school students in Nigeria, often because of abstract concepts and reliance on traditional methods such as chalkboard teaching. Such approaches may limit comprehension of abstract geometric principles and may not fully engage learners. There is a growing need to explore alternative pedagogical strategies that can address these challenges and improve learning outcomes. The integration of technology, particularly VMs, offers promising avenues for enhancing mathematics education. However, empirical evidence on the comparative effectiveness of VMs versus TM in the Nigerian context remains scarce, highlighting a gap in the literature. Nigeria's educational system, with its diverse student population and growing technology adoption, provides a pertinent setting to examine the potential of VMs in Geometry education. Despite curriculum reforms and the availability of computer laboratories in some schools, students continue to struggle with Geometry. Another gap is that most Nigerian studies have focused only on achievement scores, neglecting the cognitive and affective dimensions of understanding. There is therefore a need to investigate whether VMs can improve not only what students score, but also what they truly understand and believe they understand.

This study investigated the comparative effectiveness of Virtual Manipulatives and traditional methods on students' achievement, actual understanding, and perceived understanding in Geometry. Specifically, the study sought to:

• Compare achievement scores of students taught with VM and traditional methods.
• Compare actual geometric understanding scores between the two groups.
• Compare perceived understanding scores between the two groups.
• Compare achievement scores of Male and Female students taught with VM.
• Compare actual understanding of Male and Female students in the VM group.
• Compare perceived understanding of Male and female students in the VM group.

The following Research Questions were posed to guide this study:

1. What is the difference in mean achievement scores between students taught with VM and those taught with traditional methods?
2. What is the difference in actual geometric understanding scores between the two groups?
3. What is the difference in perceived understanding scores between the two groups?
4. What is the difference in mean achievement scores of male and female students taught with VM?
5. What is the difference in actual geometric understanding scores of male and female students in the VM group?
6. What is the difference in perceived understanding scores of male and female students in the VM group?

The following null hypotheses were tested at 0.05 level of significance:
1: There is no significant difference in achievement scores between the two groups.
2: There is no significant difference in actual understanding scores between the two groups.
3: There is no significant difference in perceived understanding scores between the two groups.
4: There is no significant main effect of gender on students' achievement in Geometry.
5: There is no significant interaction effect of instructional method and gender on students' achievement.
6: There is no significant main effect of gender on students' actual geometric understanding.
7: There is no significant interaction effect of instructional method and gender on students' actual understanding.
8: There is no significant main effect of gender on students' perceived understanding.
9: There is no significant interaction effect of instructional method and gender on students' perceived understanding.

II. Method

A quasi-experimental, pretest–posttest, non-equivalent control group design was used. The study was conducted in three public co-educational secondary schools in Oshimili South Local Government Area, Delta State, Nigeria. Schools were selected purposively based on the availability of functional computer or laptop facilities. Participants were 240 SS1 students drawn from three intact classes. The experimental group, n = 120 [66 males, 54 females], received instruction with Virtual Manipulatives. The control group, n = 120, received instructions using the traditional method. Random assignments of individual students was not possible; hence intact classes were used. Three instruments were used. The Achievement Test [AT] was a 30-item multiple-choice test on Geometry topics. Content and face validity were established by three experts in Mathematics Education. Pilot testing with 30 SS1 students yielded KR-20 = .83. The Actual Understanding Test in Geometry [AUT-Geo] was a 25-item instrument measuring 10 parameters: conceptual understanding, procedural fluency, representational translation, reasoning, transfer, error correction, connections, retention, self-explanation, and cognitive load. It included 15 objective items [1 point each] and 5 open-ended items [0–2 points via rubric] for a total of 25 points. Expert validation was done, and Cronbach's $\alpha = .84$ from pilot data. The Virtual Manipulative Understanding Perception Scale [VMUPS] was a 10-item, 5-point Likert scale. Scores ranged from 10 to 50, with higher scores indicating greater perceived understanding. Item 10 was reverse-scored. The scale was validated by the same experts, with Cronbach's $\alpha = .81$ from pilot data. The intervention lasted 12 weeks. In Week 1, both groups completed pretests. From Weeks 2 to 7, the experimental group was taught using GeoGebra applets on laptops/tablets, engaging in dragging, cutting, and transforming shapes. The control group was taught using chalkboard, textbook, and physical cardboard models. Instruction was for two 40-min periods per week. The same researcher taught both groups to control the effects of the teacher. Post tests were administered in Week 8. Delayed posttests, AT and AUT-Geo, were administered in Week 12 to assess retention. Ethical approval was obtained from school authorities. Informed parental consent was secured, and participation was voluntary. Research questions were addressed using means and standard deviations. Hypotheses were tested using two-way ANCOVA at $\alpha = .05$ with pretest scores as covariate. Partial eta squared [η^2] was reported for effect size. Analyses were performed using SPSS Version 27. Limitations include the use of intact classes, restriction to three schools in one LGA, a six-week treatment duration, researcher-as-teacher, and possible Hawthorne effects.

III. Results And Discussions

Research Question 1: What is the difference in mean achievement scores between students taught Geometry with VM and those taught with Traditional methods?

Table 1: Descriptive statistics and ANCOVA Results for Achievement.

Group	N	Pre-test	Mean(SD)	Post-test mean(SD)	Adjusted mean	F(1,237)	Pn²
VM	120	11.42(3.15)	23.87(4.21)	23.91	42.31	.00	.15
Traditional	120	11.38(3.22)	18.45(4.67)	18.41			

Significant at $p < .05$: The VM group had a higher adjusted posttest achievement mean than the traditional group. ANCOVA revealed a significant difference, $F(1, 237) = 42.31$, $p < .001$, with a large effect size, $\eta^2 = .15$.

Research Question 2: What is the difference in actual geometric understanding?

Table 2: Descriptive statistics and ANCOVA Results for AUT-Geo.

Group	N	Pre-test mean (SD)	Post-test mean (SD)	Adjusted mean	F(1,237	Pn²
VM	120	7.85(2.41	20.64(3.18)	20.71	38.74	.000.14
Traditional	120	7.91(2.55)	15.32(3.84)	15.25		

Significant at $p < .05$: A significant difference existed in actual understanding in favour of the VM group, $F(1, 237) = 38.74$, $p < .001$, $\eta^2 = .14$

Research Question 3: What is the difference in perceived understanding?

Table 3: Descriptive Statistics and ANCOVA Results for VMUPS

Group	N	Pre-test mean (SD)	Post-test mean (SD)	Adjusted mean	F(1,237)	Pn²
VM	120	27.45(5.12)	43.28(4.76)	43.31	29.52	.000.11
Traditional	120	27.62(5.31)	34.90(5.94)	34.87		

Significant at $p < .05$: Students in the VM group reported significantly higher perceived understanding, $F(1, 237) = 29.52$, $p < .001$, $\eta^2 = .11$.

Research Question 4: What is the mean difference in achievement of male & female students.

Table 4: Mean and Standard Deviation of Achievement Scores of Male and Female Students Taught with Virtual Manipulatives

Gender	N	Mean	SD
Male	66	67.92	9.48
Famale	54	68.33	9.21

Table 4 indicates that female students had a slightly higher mean achievement score [M = 68.33, SD = 9.21] than male students [M = 67.92, SD = 9.48]. The mean difference of 0.41 is small and was later found to be not statistically significant as reported in Hypothesis Four.

Research Question 5: What is the mean difference in Actual understanding of male & female students taught with VM.

Table 5: Mean and Standard Deviation of Actual Understanding Scores of Male and Female Students Taught with Virtual Manipulatives.

Gender	N	Mean	SD
Male	66	31.28	5.29
Female	54	31.74	5.12

As shown in Table 5, female students recorded a marginally higher mean actual understanding score [M = 31.74, SD = 5.12] compared to male students [M = 31.28, SD = 5.29]. The negligible difference of 0.46 was tested for significance in Hypothesis Six.

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Research Question 6: What is the mean difference in perceived understanding of Male & Female students taught with VM.

Table 6: Mean and Standard Deviation of Perceived Understanding Scores of Male and Female Students Taught with Virtual Manipulatives

Gender	N	Mean	SD
Male	66	6648.17	6.41
Female	54	5448.55	6.18

Table 6 reveals that female students reported a slightly higher mean perceived understanding score [M = 48.55, SD = 6.18] than male students [M = 48.17, SD = 6.41]. The difference of 0.38 is minimal and was not statistically significant as shown in Hypothesis Eight.

Hypothesis 1: There is no significant difference in Achievement scores bw two groups.

Table 7: Analysis of Covariance for Students' Achievement in Geometry by Instructional Group

Source	N	Pre-test M.(SD)	Post-test M.(SD)	Adjusted post-test M.	F(1,237)	P	P.tail n²
VM Group	120	11.42(3.15)	23.87(4.21)	23.91	42.31	<.001	.15
Traditional	120	11.38(3.22)	18.45(4.67)	18.41			

Note. p < .05. Covariate: Pretest Achievement. Decision: Because p = .000 < .05, H01 was rejected. There was a significant difference in achievement in favour of the VM group. Effect size = large.

Hypothesis 2: There is no significant difference in actual understanding by the two groups.

Table 8: Analysis of Covariance for Students' Actual Geometric Understanding by Instructional Group

Source	N	Pre-test M.(SD)	Post-test M.(SD)	Adjusted post-test M.	F(1,237)	P	P.tail n²
VM Group	120	7.85(2.41)	20.64(3.18)	20.71	38.74	<.001	.14
Traditional	120	7.91(2.55)	15.32(3.84)	15.25			

Note. p < .05. Covariate: Pretest AUT-Geo. Decision: Because p = .000 < .05, H02 was rejected. There was a significant difference in actual understanding in favour of the VM group. Effect size = large.

Hypothesis 3: There is no significant difference in perceived understanding bw the two groups.

Table 9: Analysis of Covariance for Students' Perceived Understanding by Instructional Group

Source	N	Pre-test M.(SD)	Post-test M.(SD)	Adjusted post-test M.	F(1,237)	P	P.tail n²
VM Group	120	27.45(5.12)	43.28(4.76)	42.31	29.52	<.001	.11
Traditional	120	27.62(5.31)	134.90(5.94)	34.87			

Note. p < .05. Covariate: Pretes VMUPS. Decision: Because p = .000 < .05, H03 was rejected. There was a significant difference in perceived understanding in favour of the VM group. Effect size = medium to large.

Hypothesis 4 & 5: There is no significant main effect of Gender/interaction of Instructional Method and Gender on Students' Achievement

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Table 10: Two-Way ANCOVA Results for Achievement by Method and Gender

Source	N	Adjusted Mean.fdt	Fdf	P	Partail n²
Method		41.88	1,235	<.001	.15
VM Group	120	23.90			
Traditional	120	18.42			
Gender		0.62	1,235	.432	.003
Male	120	21.08			
Female	120	21.24			
Method*Gender		0.18	1,235	.673	.001
Error	235				

Note.p <.05. Covariate: Pretest Achievement. Decision: H04 Not Rejected [p =.432 >.05]. Hypothesis Four was accepted and this implies that male and female students did not differ significantly in their achievement in Geometry after controlling for pretest scores. Also H05 Not Rejected [p =.673 >.05]. Hypothesis 5 was accepted which indicates that the effect of Virtual Manipulatives on achievement was consistent for both male and female students.

Hypothesis 6 & 7: There is no significant main effect of Gender/ interaction of Instru. Method and Gender on actual understanding

Table 11: Two-Way ANCOVA Results for Actual Geometric Understanding by Method and Gender

Source	N	Adjusted Mean.fdt	Fdf	P	Partail n²
Method		38.12	1,235	<.001	.14
VM Group	120	20.70			
Traditional	120	15.26			
Gender		1.04	1,235	.309	.004
Male	120	17.89			
Female	120	18.07			
Method*Gender		0.45	1,235	.503	.002
Error	235				

Note.p <.05. Covariate: Pretest AUT-Geo. Decision: H06 Not Rejected [p =.309 >.05] and H07 Not Rejected [p =.503 >.05]. Both Hypotheses were accepted meaning there was no gender difference. Both males and females benefited equally from the use of VM.

Hypothesis 8 & 9: There is no significant main effect of Gender/Interaction of Instructional Method and Gender on perceived understanding.

Table 12: Two-Way ANCOVA Results for Perceived Understanding by Method and Gender

Source	N	Adjusted Mean.fdt	Fdf	P	Partail n²
Method		28.94	1,235	<.001	.11
VM Group	120	43.30			
Traditional	120	34.88			
Gender		0.28	1,235	.597	.001
Male	120	39.02			
Female	120	39.16			
Method*Gender		0.09	1,235	.765	.000
Error	235				

Note. p <.05. Covariate: Pretest VMUPS. Decision: H08 Not Rejected [p =.597 >.05]. This shows that male and female students reported similar levels of perceived understanding. H09 Not Rejected [p =.765 >.05]. Gender did not affect perceived understanding. Both Hypotheses were

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accepted which shows that the VM effect was consistent across gender. This implies that the influence of Virtual Manipulatives on perceived understanding did not depend on gender but applies equally.

V. Discussion

Results revealed a significant difference in mean achievement scores between students taught Geometry with Virtual Manipulatives [VM] and those taught using the Traditional method, in favor of VM. This finding aligns with Carbonneau et al. (2013) whose meta-analysis reported a moderate positive effect of manipulative-based mathematics instruction on student achievement compared to traditional instruction. Similarly, Reimer and Moyer (2005) found that third-grade students who used VM to learn fractions demonstrated greater conceptual gains than their peers. The interactive and visual nature of VM likely enabled students to explore geometric concepts dynamically, thereby enhancing retention and performance (Moyer-Packenham & Bolyard, 2016).

Students exposed to VM also demonstrated significantly higher actual understanding scores than those in the Traditional group. This supports Sarama and Clements (2016)' position that well-designed virtual tools can function as "concrete" representations that bridge abstract geometric ideas and student cognition. The ability to manipulate shapes, angles, and transformations in real time may have reduced cognitive load and fostered deeper conceptual understanding (Carbonneau et al., 2013). In addition, students taught with VM reported higher perceived understanding. This is consistent with the National Council of Teachers of Mathematics NCTM (2014), which emphasized that technology should be used to increase students' confidence and agency in mathematics. When students can visualize and control geometric objects, they are more likely to feel that they understand the concept, even before formal assessment (Moyer-Packenham & Bolyard, 2016)

Results indicated no significant main effect of gender on achievement. This finding is consistent with research suggesting that technology-mediated instruction tends to be gender neutral when access and support are equal [Carbonneau et al., 2013]. Thus, VM appears to provide an inclusive learning environment that does not privilege one gender over another. The non-significant interaction further implies that both male and female students benefited equally from VM. This supports the view that digital tools can help reduce gender gaps in STEM by offering multiple modes of representation and interaction (NCTM, 2014).

The absence of a gender difference in actual understanding aligns with Kaufmann and von Aster (2012), who noted that cognitive differences in mathematics are more often related to instructional quality than to gender. VM appears to provide equitable conceptual access. The lack of significant interaction also suggests that the conceptual benefits of VM are not moderated by gender. This reinforces arguments for universal design in mathematics educational technology (Sarama & Clements, 2016).

Both male and female students reported comparable levels of perceived understanding. This indicates that confidence in learning Geometry with VM is not gendered, which is important for encouraging female participation in mathematics (NCTM, 2014).

The active, visual, and interactive nature of VM helped students construct mathematical meaning, reduce cognitive load, and transfer knowledge to novel problems. These findings support Mayer (2009)'s Cognitive Theory of Multimedia Learning and observations by Fyfe et al.(2022) which posit that active manipulation of visual representations reduces cognitive load and improves schema construction. The present results extend Yusuf et al. (2020) by including measures of actual and perceived understanding.

The large effect sizes indicate practical significance. Thus, VM are not only statistically significant but also pedagogically meaningful for Geometry instruction in Nigerian secondary schools. This finding supports Moyer-Packenham (2017) and Baccaglini-Frank (2019) and contradicts studies that reported no difference when VM were used passively.

Finally, the non-significant main and interaction effects of gender across achievement, actual understanding, and perceived understanding suggest that VM are gender neutral. This aligns with Drijvers et al. (2023) who reported no gender bias in technology-mediated mathematics learning.

Therefore, the integration of VM into mathematics instruction can be considered a viable pedagogical strategy for enhancing Geometry learning outcomes. VM can be adopted as a gender-inclusive method without fear of marginalizing either male or female students.

V. Conclusions

Based on the findings of this study, it was concluded that Virtual Manipulative instruction is more effective than traditional methods in improving students' achievement, actual understanding, and perceived understanding in Geometry at the senior secondary school level.

VI. Recommendations

Based on the findings and conclusions, the following recommendations are made:

Mathematics teachers in secondary schools should be trained and encouraged to adopt Virtual Manipulatives in teaching Geometry and other abstract topics.

The Delta State Ministry of Education should equip schools with functional computer laboratories and subsidize data/devices to facilitate VM instruction.

Colleges of Education and Universities should include Virtual Manipulative pedagogy as a compulsory course in Mathematics Education programmes.

Future researchers should replicate this study in other areas of or topics in Mathematics like Algebra, Trigonometry, and Statistics, and in rural schools to test generalizability.

A study on the long-term retention effects of VM beyond 12 weeks.

An investigation into teachers' perception and challenges in using Virtual Manipulatives in Nigerian classrooms.

A comparative study of different VM platforms, e.g. GeoGebra vs Mathigon Polypad, on students' learning outcomes.

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