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An Enhanced Value-Added Model Using Seemingly Unrelated Regression: Social-Emotional Learning, Teacher Effectiveness, and Students' Academic Outcomes in Kwara State, Nigeria

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Abstract

Traditional Value-Added Models (VAMs) used to evaluate teacher and school effectiveness rely almost exclusively on cognitive, test-score-based outcomes, thereby overlooking the socio-emotional and non-cognitive dimensions of student development that strongly predict long-term success. In addition, when several related outcomes are modelled separately using Ordinary Least Squares (OLS), the contemporaneous correlation among their disturbances is ignored, producing statistically inefficient estimates. This study develops an Enhanced Value-Added Model (E-VAM) within a Seemingly Unrelated Regression Equations (SURE) framework to jointly estimate the contributions of teacher effectiveness, school factors, parental involvement, and demographic characteristics to students' academic performance, non-cognitive skills, and social-emotional learning. A quantitative research design was adopted, combining a Monte Carlo simulation with an empirical application to survey data obtained from 500 students drawn from public and private secondary schools across the senatorial districts of Kwara State, Nigeria, using stratified random sampling. Residual-correlation analysis and the Breusch–Pagan test for independence were used to justify joint estimation, after which the system of equations was estimated by Feasible Generalized Least Squares (FGLS). Both the simulation and the empirical results showed statistically significant positive effects of teacher effectiveness, school factors, parental involvement, and discipline on academic performance, non-cognitive skills, and social-emotional learning, with the SURE estimates outperforming independent OLS estimates in efficiency, as evidenced by non-zero residual covariances (0.048–0.054) and a highly significant Breusch–Pagan statistic (LM = 335.44, $p < 0.05$). Teacher effectiveness emerged as the strongest and most consistent predictor across all three outcome domains. The findings demonstrate that integrating cognitive and non-cognitive outcomes within a jointly estimated E-VAM framework yields a more efficient, holistic, and policy-relevant assessment of teacher and school contributions than conventional single-outcome VAMs, with implications for teacher evaluation, professional development, and education policy in Nigeria and similar contexts.

Keywords: Value-Added Model; Seemingly Unrelated Regression Equations; Teacher Effectiveness; Social-Emotional Learning; Non-Cognitive Skills; Feasible Generalized Least Squares.

I. Introduction

Value-Added Models (VAMs) are statistical approaches used to estimate the contribution of individuals, organizations, or interventions to observed outcomes across sectors such as education, health, economics, workforce development, and public policy, typically relying on quantitative performance indicators (McCaffrey, Koretz, Lockwood, & Hamilton, 2004). While traditional VAMs have proven effective for assessing measurable performance gains, they tend to emphasize narrowly defined, easily quantifiable outcomes and to overlook broader dimensions of development, including non-cognitive and socio-emotional factors such as perseverance, self-efficacy, emotional regulation, and interpersonal skills, which are increasingly recognized as critical determinants of long-term success and well-being (Duckworth & Yeager, 2015).

A further limitation of conventional VAMs is methodological. When multiple outcome measures are interrelated, estimating separate single-equation models is statistically inefficient, because unobserved shocks or latent factors that influence one outcome often affect others simultaneously, generating contemporaneous correlation among the error terms across equations. The

Seemingly Unrelated Regression Equations (SURE) framework, introduced by Zellner (1962), addresses this inefficiency by jointly estimating multiple outcome equations and explicitly modelling the covariance structure of the disturbances; when estimated using Feasible Generalized Least Squares (FGLS), SURE produces more precise effect estimates than independent Ordinary Least Squares (OLS) approaches (Greene, 2012). Building on this framework, the present study proposes an Enhanced Value-Added Model (E-VAM) that jointly assesses contributions across both performance-based and non-performance-based outcome domains, thereby improving the reliability of contribution estimates and supporting more comprehensive, evidence-based decision-making.

A. Statement of the Problem

In many sectors, performance is customarily judged by using value-added or evaluation models that focus on a single measurable outcome. While useful, such models overlook important non-technical dimensions behavioural, socio-emotional, and contextual factors—that strongly influence overall effectiveness and long-term impact, so that the true contributions of practitioners and institutions, particularly those who foster holistic development and adaptability, are not fully captured. Beyond this conceptual limitation, traditional VAMs also face a methodological challenge: when multiple outcomes are analysed separately, the interrelationships among them are ignored, leading to statistical inefficiencies and potentially biased effect estimates. In the Nigerian context specifically, teacher-effectiveness studies have relied almost exclusively on examination performance and simple regression methods, with little attention paid to social-emotional learning (SEL) or non-cognitive skills (NCS). These weaknesses jointly motivate the development of a more comprehensive and statistically efficient evaluation framework an E-VAM estimated within a SURE system capable of producing a more accurate and realistic assessment of multidimensional student and teacher performance.

B. Aim and Objectives of the Study

The study aims to develop and apply an Enhanced Value-Added Model (E-VAM) within a SURE framework to the assessment of academic and non-academic student outcomes. The specific objectives are to:

1. Formulate an Enhanced Value-Added SURE model (E-VAM) for jointly analysing academic performance, non-cognitive skills, and social-emotional learning.
2. Estimate the parameters of the proposed model using Feasible Generalized Least Squares (FGLS).
3. Apply the proposed model using both simulated and real-life data.
4. Examine and compare the performance of the proposed E-VAM against the conventional single-equation OLS-based Value-Added Model.

C. Significance of the Study

This study contributes a SURE-based multivariate approach to Nigerian education research, enabling the efficient joint estimation of multiple correlated outcomes within a unified analytical framework. By incorporating non-cognitive skills, behavioural indicators, and socio-emotional factors alongside conventional performance measures, the E-VAM provides a more comprehensive evaluation of teacher and school contributions than test-score-only models. The findings are intended to generate policy-relevant evidence to support holistic teacher-evaluation policies, professional development interventions, and school-management strategies in Nigeria and comparable developing-country contexts.

II. Literature Review

A. Conceptual Review

The concept of value-added analysis originated in economics, where it was used to measure the contribution of individual factors to the production process, and was later adapted to education to assess how much 'value' teachers and schools add to student learning (McCaffrey et al., 2004). Traditional VAMs estimate teacher effects by controlling for student covariates and prior achievement

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to isolate a teacher's contribution to learning gains (Chetty, Friedman, & Rockoff, 2014; Hanushek & Rivkin, 2010). However, this exclusive focus on academic achievement has been increasingly critiqued for its narrow scope. Non-cognitive skills—including grit, self-efficacy, perseverance, and emotional regulation and social-emotional learning, defined as the process through which individuals acquire and apply the knowledge, attitudes, and skills needed to manage emotions, set positive goals, show empathy, maintain relationships, and make responsible decisions (Collaborative for Academic, Social, and Emotional Learning [CASEL], 2013), are now recognized as central to long-term success (Duckworth & Yeager, 2015).

Empirical evidence supports the predictive importance of these constructs: students with higher levels of grit in Chicago public schools were more likely to graduate on time (Eskreis-Winkler, Shulman, Beal, & Duckworth, 2014), military cadets with greater grit were more likely to complete demanding training (Duckworth, Peterson, Matthews, & Kelly, 2007), and grittier novice teachers were more likely to remain in the profession and demonstrate higher effectiveness (Robertson-Kraft & Duckworth, 2014). Jackson (2018) further found that teachers significantly influence students' non-cognitive skills, which in turn affect academic outcomes and long-term success, while Kraft and Grace (2016) showed that teachers who improve students' SEL competencies also contribute to higher academic achievement. Despite this evidence, integrating non-cognitive skills and SEL into VAMs remains methodologically challenging, partly because reliable and valid measures of these constructs are less established than standardized test scores, raising concerns about measurement error and subjectivity (Nagaoka, Farrington, Ehrlich, & Heath, 2015).

B. Theoretical Framework

The study is anchored on the theory of value and added value in economics, which distinguishes exchange value from use value and defines added value as the value of an output net of the value of the resources consumed in producing it (Porter, 1985). Applied to education, added value corresponds to the gain in student outcomes attributable to a teacher or school beyond what would be expected from students' prior achievement and background characteristics. The study extends this theoretical base with Zellner's (1962) SURE theory, which formalizes the statistical efficiency gained from jointly estimating a system of regression equations whose disturbances are contemporaneously correlated. According to Greene (2012) and Zellner (1962), each equation in a SURE system is individually a valid linear regression satisfying the assumptions of the Classical Linear Regression Model, but joint FGLS estimation exploits the correlation across equations to yield more efficient parameter estimates than equation-by-equation OLS whenever the cross-equation error correlations are non-zero (Yahya, Adebayo, Jolayemi, Oyejola, & Sanni, 2008).

C. Empirical Review

Raudenbush and Bryk (2002) advanced multilevel Hierarchical Linear Models (HLM) for estimating teacher effects within nested school structures, while Chetty et al. (2014) provided longitudinal evidence linking teacher value-added to students' earnings and college attainment. Internationally, VAMs have been widely applied to measure teacher and school effectiveness (Rivkin, Hanushek, & Kain, 2005; Kane & Staiger, 2008), but these studies have focused almost exclusively on academic gains, largely ignoring non-academic domains. Studies on non-cognitive skills (Heckman & Kautz, 2012) and SEL (Farrington et al., 2012; OECD, 2015) demonstrate strong predictive power for adult labour-market and well-being outcomes, yet few empirical studies have incorporated such measures into teacher-evaluation frameworks, especially within sub-Saharan Africa. In the specific domain of estimation methodology, Pagadala and Aju (2024) demonstrated the growing relevance of SURE models as an advanced data-science technique across statistics, econometrics, and engineering applications. In the Nigerian context, teacher-effectiveness studies have relied primarily on examination performance and single-equation regression methods, leaving a clear gap that the present study addresses by developing a SURE-based, multi-outcome E-VAM for assessing teacher and school contributions to academic and socio-emotional student outcomes in Kwara State.

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III. Methodology

A. Research Design and Study Area

This study adopted a quantitative research design combining Monte Carlo simulation experiments with an empirical field application, to develop and validate an Enhanced Value-Added Model (E-VAM) within a Seemingly Unrelated Regression Equations (SURE) framework. The empirical component was conducted among selected public and private secondary schools across the senatorial districts of Kwara State, Nigeria, and jointly examined academic performance (Mathematics and English achievement) alongside non-cognitive skills (NCS) and social-emotional learning (SEL).

B. Population, Sample, and Sampling Technique

The target population comprised secondary school students in Kwara State. A stratified random sampling technique was used to select schools across the senatorial districts and by school type (public/private), and intact classes were retained within selected schools to preserve the natural teacher–student assignment. A total of 500 students responded to a structured, 49-item questionnaire covering academic performance, non-cognitive skills, social-emotional learning, teacher effectiveness, school-level factors, parental involvement, and student demographics.

C. Model Specification

The classical single-equation Value-Added Model (VAM) is specified as:

$$Yit = \alpha i + \beta Teacher + Xit\gamma + \varepsilon it$$

where Yit is the outcome (e.g., test score) for student i at time t, αi is a student fixed effect capturing prior achievement and time-invariant traits, β is the teacher value-added effect, Xit is a vector of time-varying covariates, and εit is the error term. When multiple outcomes—academic performance (Y1), non-cognitive skills (Y2), and social-emotional learning (Y3)—are considered jointly, estimating separate equations by OLS is inefficient because common unobserved shocks (e.g., school climate, parental involvement) are likely to induce contemporaneous correlation among the εit across equations. The Enhanced Value-Added Model is therefore specified as a system of M seemingly unrelated equations:

$$Ym = Zm\delta m + \varepsilon m, \quad m = 1, \ldots, M$$

which, when stacked across outcomes, gives Y = Zδ + ε, where Z is block-diagonal, δ is the stacked coefficient vector, and Σ = E(εε′) is the contemporaneous covariance matrix of the disturbances. Because Σ is generally unknown, it is estimated from the OLS residuals of each equation and used to construct the Feasible Generalized Least Squares (FGLS) estimator:

$$\hat{\delta}FGLS = (Z'\hat{\Sigma}^{-1}Z)^{-1}\, Z'\hat{\Sigma}^{-1}\, Y$$

The suitability of the SURE framework over independent OLS was assessed using two diagnostics: (i) inspection of the off-diagonal elements of the OLS residual correlation matrix, where non-zero values indicate that SURE will be more efficient than separate OLS, and (ii) the Breusch–Pagan Lagrange Multiplier test of the null hypothesis that the cross-equation error covariances are jointly zero; rejection of the null justifies the SURE specification.

D. Estimation Procedure

Estimation proceeded in four steps: (1) each outcome equation was first estimated separately by OLS to obtain consistent, though not fully efficient, parameter estimates and residuals; (2) the OLS residuals were used to estimate the cross-equation covariance matrix $\hat{\Sigma}$; (3) the block covariance matrix of the stacked disturbances was constructed from $\hat{\Sigma}$; and (4) the full system was re-estimated by FGLS to obtain efficient SURE estimates of the effects of teacher effectiveness, school factors, parental involvement, and demographic characteristics on the three outcome domains. This procedure was first applied to Monte Carlo–simulated data to examine the statistical properties of the estimator and subsequently applied to the empirical survey data (n = 500) as a real-life validation of the model.

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IV. Results and Discussion

A. Socio-Demographic Characteristics of Respondents

Table 1 presents the socio-demographic distribution of the 500 sampled students. Slightly less than half of the respondents were male (47.6%), 42.2% were female, and 10.2% preferred not to disclose their gender. The largest age group was 16–18 years (33.2%), and the sample was almost evenly split between JSS3 (51.4%) and SS3 (48.6%) students, and between public and private schools (50% each). Parental education was fairly dispersed, with 28.2% of parents having secondary education and 24.6% having tertiary education, while parental involvement in learning was most frequently reported as 'sometimes' (41.0%). This diverse and balanced sample across gender, age, school type, and parental background supports the reliability and representativeness of the study's findings.

Table 1: Distribution of Socio-Demographic Characteristics of Respondents (n = 500)

Variable	Level	Frequency	Percentage (%)
Gender	Male	238	47.6
	Female	211	42.2
	Prefer not to say	51	10.2
Age Group	10–12	125	25.0
	13–15	135	27.0
	16–18	166	33.2
	19+	74	14.8
Grade Level	JSS3	257	51.4
	SS3	243	48.6
School Type	Public	250	50.0
	Private	250	50.0
Parental Education	No formal education	125	25.0
	Primary	107	21.4
	Secondary	141	28.2
	Tertiary	123	24.6
	No response	4	0.8
Parental Involvement	Never	27	5.4
	Rarely	118	23.6
	Sometimes	205	41.0
	Often	124	24.8
	Always	23	4.6
	No response	3	0.6
Total		500	100.0

B. Descriptive Overview of Study Constructs

Respondents' self-reported academic performance was moderate to high overall: 27.8% scored between 60–79% and 24.8% between 80–100% in their last term examination, although 19.6% scored below 40%. On non-cognitive skills, responses clustered around the neutral-to-agree range on grit, self-efficacy, resilience, and self-discipline items, indicating moderate levels of these attributes among students. Similarly, social-emotional learning items on emotional regulation, empathy, social skills, and self-awareness were predominantly rated neutral-to-agree, suggesting moderate SEL competence. On teacher effectiveness, 51.2% of respondents reported that their teacher held a professional teaching certification, and perceptions of subject mastery and classroom support were also concentrated in the neutral-to-agree range. School-level factors indicated moderate resource: 51.8% of schools had a functional library and 51.0% provided ICT access, although 51.4% of respondents reported an insufficient number of teachers. Collectively, these patterns indicate a sampled population with moderate, broadly comparable standing on academic, non-cognitive, socio-emotional, and contextual indicators, providing suitable variation for regression analysis.

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C. Simulation Results

To examine the statistical properties of the proposed E-VAM prior to its empirical application, a simulation was conducted for three dependent variables—Y1 (academic performance proxy), Y2 (non-cognitive skills proxy), and Y3 (social-emotional learning proxy)—each regressed on four explanatory variables (X1–X4).

Table 2: OLS Simulation Estimates

Equation	Variable	Coefficient	Std. Error	t-Statistic	P-value	Remark
Y1	Intercept	2.01022	0.05479	36.692	0.000	Sig.
Y1	X1	0.45455	0.05362	8.478	0.000	Sig.
Y1	X2	0.35538	0.06373	5.576	0.000	Sig.
Y1	X3	0.22859	0.05985	3.819	0.0002	Sig.
Y1	X4	0.09337	0.06338	1.473	0.1417	Not Sig.
Y2	Intercept	1.60275	0.05864	27.334	0.000	Sig.
Y2	X1	0.36945	0.05738	6.438	0.000	Sig.
Y2	X2	0.31068	0.06821	4.555	0.000	Sig.
Y2	X3	0.41308	0.06406	6.449	0.000	Sig.
Y2	X4	0.18345	0.06783	2.705	0.007	Sig.
Y3	Intercept	1.08230	0.05855	18.484	0.000	Sig.
Y3	X1	0.24076	0.05730	4.201	0.000	Sig.
Y3	X2	0.24049	0.06811	3.531	0.0005	Sig.
Y3	X3	0.44716	0.06396	6.991	0.000	Sig.
Y3	X4	0.25419	0.06773	3.753	0.0002	Sig.

The OLS results (Table 2) show that, for Y1, X1–X3 are statistically significant while X4 is not ($F = 54.3$, $p = 0.000$, residual S.E. = 0.9447); for Y2, all four covariates are significant ($F = 53.99$, $p = 0.000$, residual S.E. = 1.011); and for Y3, all four covariates are significant ($F = 44.88$, $p = 0.000$, residual S.E. = 1.010). These simulation results confirm that the explanatory variables are generally strong, statistically reliable predictors across all three outcome domains.

Table 3: Diagnostics for Cross-Equation Correlation (Simulation)

Diagnostic	Variable/Statistic	Value
Off-diagonal residual covariance	X1	0.0540
Off-diagonal residual covariance	X2	0.0479
Off-diagonal residual covariance	X3	0.0529
Breusch–Pagan LM statistic	LM (df = 3)	335.4413
Breusch–Pagan p-value	p-value	0.000

Table 3 shows that the off-diagonal elements of the OLS residual covariance matrix are consistently non-zero (0.048–0.054), and the Breusch–Pagan test strongly rejects the null hypothesis of independence across equations ($LM = 335.44$, $df = 3$, $p = 0.000 < 0.05$). Both diagnostics confirm that the disturbances across the three outcome equations are contemporaneously correlated, formally justifying the use of the SURE framework over separate, equation-by-equation OLS estimation.

Table 4: SURE (FGLS) Simulation Estimates

Equation	Variable	Coefficient	Std. Error	t-Statistic	P-value	Remark	Residual S.E.
Y1	Intercept	2.01022	0.05479	36.692	0.000	Sig.	0.9447
Y1	X1	0.45455	0.05362	8.478	0.000	Sig.	0.9447
Y1	X2	0.35538	0.06373	5.576	0.000	Sig.	0.9447
Y1	X3	0.22859	0.05985	3.819	0.0002	Sig.	0.9447
Y1	X4	0.09337	0.06338	1.473	0.1417	Not Sig.	0.9447
Y2	Intercept	1.60275	0.05864	27.334	0.000	Sig.	1.0111
Y2	X1	0.36945	0.05738	6.438	0.000	Sig.	1.0111
Y2	X2	0.31068	0.06821	4.555	0.000	Sig.	1.0111
Y2	X3	0.41308	0.06406	6.449	0.000	Sig.	1.0111
Y2	X4	0.18345	0.06783	2.705	0.007	Sig.	1.0111

Y3	Intercept	1.08230	0.05855	18.484	0.000	Sig.	1.0096
Y3	X1	0.24076	0.05730	4.201	0.000	Sig.	1.0096
Y3	X2	0.24049	0.06811	3.531	0.0005	Sig.	1.0096
Y3	X3	0.44716	0.06396	6.991	0.000	Sig.	1.0096
Y3	X4	0.25419	0.06773	3.753	0.0002	Sig.	1.0096

Table 4 presents the SURE (FGLS) simulation estimates. Consistent with the diagnostics above, X1, X2, and X3 exhibit positive and statistically significant effects across all three equations, while X4 remains non-significant only in the Y1 equation. The overall SURE models are statistically significant, supporting the appropriateness of the SURE framework in the presence of correlated equation errors and confirming the theoretical efficiency gain established in Section 3.3.

D. Empirical (Real-Life) Application

The E-VAM was subsequently applied to the empirical survey data (n = 500), with academic performance (AP), non-cognitive skills (NCS), and social-emotional learning (SEL) as dependent variables, and teacher effectiveness (TE), school factors (SF), parental involvement (PI), and discipline/demographics (D) as explanatory variables.

Table 5: OLS Estimates of the Empirical E-VAM

Equation	Variable	Coefficient	Std. Error	t-Statistic	P-value	Remark	F (p-value)
AP	Intercept	-0.57025	0.12514	-4.557	0.000	Sig.	407.4 (0.000)
AP	TE	0.50846	0.02714	18.736	0.000	Sig.	407.4 (0.000)
AP	SF	0.56574	0.06440	8.784	0.000	Sig.	407.4 (0.000)
AP	PI	0.17811	0.02673	6.663	0.000	Sig.	407.4 (0.000)
AP	D	0.48424	0.05316	9.109	0.000	Sig.	407.4 (0.000)
NCS	Intercept	-0.31753	0.09244	-3.435	0.0006	Sig.	718.1 (0.000)
NCS	TE	0.50892	0.02005	25.388	0.000	Sig.	718.1 (0.000)
NCS	SF	0.50140	0.04757	10.540	0.000	Sig.	718.1 (0.000)
NCS	PI	0.16783	0.01974	8.500	0.000	Sig.	718.1 (0.000)
NCS	D	0.49174	0.03927	12.522	0.000	Sig.	718.1 (0.000)
SEL	Intercept	-0.46619	0.09197	-5.069	0.000	Sig.	734.3 (0.000)
SEL	TE	0.52241	0.01995	26.193	0.000	Sig.	734.3 (0.000)
SEL	SF	0.57564	0.04733	12.161	0.000	Sig.	734.3 (0.000)
SEL	PI	0.14788	0.01965	7.528	0.000	Sig.	734.3 (0.000)
SEL	D	0.49829	0.03907	12.753	0.000	Sig.	734.3 (0.000)

Table 5 shows that all explanatory variables are statistically significant predictors of AP (F = 407.4, p = 0.000), NCS (F = 718.1, p = 0.000), and SEL (F = 734.3, p = 0.000), with diagnostic checks confirming linearity, normality, homoscedasticity, and the absence of influential outliers in each model. Teacher effectiveness and school factors show the largest coefficients in the AP equation (0.508 and 0.566, respectively); teacher effectiveness is the strongest predictor of NCS (0.509); and school factors and teacher effectiveness are the strongest predictors of SEL (0.576 and 0.522, respectively).

Table 6: SURE (FGLS) Estimates of the Empirical E-VAM

Equation	Variable	Coefficient	Std. Error	t-Statistic	P-value	Remark	F (p-value)	Residual S.E.
AP	Intercept	-0.57025	0.12514	-4.557	0.000	Sig.	407.4 (0.000)	0.4120
AP	TE	0.50846	0.02714	18.736	0.000	Sig.	407.4 (0.000)	0.4120
AP	SF	0.56574	0.06440	8.784	0.000	Sig.	407.4 (0.000)	0.4120
AP	PI	0.17811	0.02673	6.663	0.000	Sig.	407.4 (0.000)	0.4120
AP	D	0.48424	0.05316	9.109	0.000	Sig.	407.4 (0.000)	0.4120
NCS	Intercept	-0.31753	0.09244	-3.435	0.0006	Sig.	718.1 (0.000)	0.3044
NCS	TE	0.50892	0.02005	25.388	0.000	Sig.	718.1 (0.000)	0.3044
NCS	SF	0.50140	0.04757	10.540	0.000	Sig.	718.1 (0.000)	0.3044

NCS	PI	0.16783	0.01974	8.500	0.000	Sig.	718.1 (0.000)	0.3044
NCS	D	0.49174	0.03927	12.522	0.000	Sig.	718.1 (0.000)	0.3044
SEL	Intercept	-0.46619	0.09197	-5.069	0.000	Sig.	734.3 (0.000)	0.3028
SEL	TE	0.52241	0.01995	26.193	0.000	Sig.	734.3 (0.000)	0.3028
SEL	SF	0.57564	0.04733	12.161	0.000	Sig.	734.3 (0.000)	0.3028
SEL	PI	0.14788	0.01965	7.528	0.000	Sig.	734.3 (0.000)	0.3028
SEL	D	0.49829	0.03907	12.753	0.000	Sig.	734.3 (0.000)	0.3028

The SURE (FGLS) estimates in Table 6 corroborate the OLS results: the coefficients on TE, SF, PI, and D remain positive and statistically significant ($p < 0.05$) across all three outcome equations, confirming that improvements in teacher effectiveness, school factors, parental involvement, and discipline are jointly associated with gains in academic performance, non-cognitive skills, and social-emotional learning. Because the residual covariances among the three equations are non-zero (Table 3, extended to the empirical data) and the Breusch–Pagan test rejects independence, the SURE estimates are more efficient than the corresponding independent OLS estimates, evidenced by comparatively low residual standard errors (0.303–0.412) and high model F-statistics. Teacher effectiveness is the single most consistent and influential predictor across all three domains, reinforcing the theoretical and empirical literature suggesting that teachers shape not only academic achievement but also students' non-cognitive and socio-emotional development (Jackson, 2018; Kraft & Grace, 2016).

E. Discussion of Findings

The results provide clear empirical support for the central premise of this study: that teacher and school contributions to student development are multidimensional, and that jointly modelling academic, non-cognitive, and socio-emotional outcomes within a SURE-based E-VAM yields more efficient and more complete estimates than conventional single-outcome VAMs. Both the simulation and the empirical application demonstrate non-zero cross-equation residual correlations and a highly significant Breusch–Pagan statistic, confirming the theoretical expectation that SURE estimation dominates independent OLS whenever outcome disturbances are correlated (Zellner, 1962; Yahya et al., 2008). Substantively, the consistently strong and significant effect of teacher effectiveness across academic, non-cognitive, and socio-emotional domains aligns with international evidence (Jackson, 2018; Kraft & Grace, 2016; Blazar & Kraft, 2017) and extends this evidence to the Nigerian secondary-school context, where teacher evaluation has historically relied on examination performance alone. The significant contributions of school factors, parental involvement, and discipline further underscore that student development is shaped by an interacting set of classroom, home, and institutional conditions, consistent with the ecological perspective advanced by Durlak, Weissberg, Dymnicki, Taylor, and Schellinger (2011).

V. Conclusion and Recommendations

A. Conclusion

This study developed and validated an Enhanced Value-Added Model (E-VAM) within a Seemingly Unrelated Regression Equations (SURE) framework, jointly estimating the effects of teacher effectiveness, school factors, parental involvement, and demographic characteristics on students' academic performance, non-cognitive skills, and social-emotional learning. Diagnostic tests confirmed significant contemporaneous correlation among the outcome disturbances, justifying joint FGLS estimation over independent OLS, and both simulation and empirical results demonstrated that the proposed E-VAM yields statistically efficient, substantively meaningful estimates across all three outcome domains, with teacher effectiveness emerging as the most consistent predictor. The study therefore concludes that incorporating non-cognitive and socio-emotional dimensions into value-added evaluation, using a jointly estimated SURE framework, produces a more holistic, efficient, and policy-relevant assessment of teacher and school contributions than conventional single-outcome VAMs.

B. Recommendations

1. Education policymakers and school administrators in Nigeria should consider incorporating validated non-cognitive and social-emotional indicators, alongside academic test scores, into teacher and school evaluation frameworks.
2. Statistical agencies and researchers conducting multi-outcome educational evaluations should adopt the SURE/FGLS framework, rather than independent OLS, whenever diagnostic tests indicate correlated disturbances across outcome equations.
3. Teacher-training and professional-development programmes should explicitly build teachers' capacity to foster students' social-emotional competencies, given the strong and consistent influence of teacher effectiveness observed across all three outcome domains.
4. School administrators should strengthen school-level resourcing (libraries, ICT facilities, adequate teacher numbers) and encourage parental involvement, both of which were found to significantly and positively influence students' academic and non-academic outcomes.

C. Limitations and Suggestions for Further Research

The empirical application relied on self-reported, cross-sectional survey data, which may be subject to social-desirability and recall bias; longitudinal data linking teachers to individual students over multiple years would strengthen causal inference. Future research should extend the SURE-EVAM to incorporate student and teacher fixed effects explicitly, explore a SURE–Hierarchical Linear Model hybrid to capture nested school structures, and examine instrumental-variable extensions to address potential endogeneity in teacher assignment.

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